A Generalization of the TSD Network-Analyzer Calibration Procedure, Covering n-Port Scattering-Parameter Measurements, Affected by Leakage Errors

ROSS A. SPECIALE

Abstract—**The basic philosophy of the through-short-delay (TSD) calibration procedure for two-port automated network analyzers has been extended to n-port scattering-parameter measurements, while also accounting for the errors due to possible signal leakage between all port pairs.**

The system errors are represented by the scattering response of a $2n$-port virtual error network, having n ports connected to the device under test and n ports connected to an ideal error-free multiport network analyzer.

The $(2n)^2$ T-parameters of the error network are explicitly expressed in blocks of n^2 at a time, as matricial functions of the $3n^2$ S-parameters of three n-port standards, sequentially replacing the device under test during system calibration.

The possibility has also been investigated of correcting the errors due to repeatable measurement-port mismatch changes, typical of switching scattering-parameter test sets. This capability has been introduced and tested in the classical two-port TSD calibration algorithm, by means of a minor modification and data postprocessing, applied after the removal of conventional errors.

I. Introduction

EVER SINCE the introduction of automated microwave instrumentation for the characterization of microwave components and networks through scattering-parameter measurements, the need was recognized for automated system-calibration procedures. These were expected to be capable of providing a representation of the repeatable system errors, usable for correcting uncalibrated measurements.

A large variety of error models and calibration procedures has been proposed to date, all differing in degree of complexity and effectiveness [1]–[18]. A common feature of all the proposed error models is the attempt at representing the repeatable system errors by means of the scattering response of a virtual error network, assumed to interface the device under test to an ideal, error-free network-analyzer system. The various proposed error models differ, however, in the assumed topological configuration of the specific error network and in the number of independent complex parameters required for its full characterization.

The removal of the computed measurement errors from uncalibrated measurements must be performed through a parameter transformation equivalent to removing (or "stripping") the virtual error network from the measurement interface. A common feature of all proposed calibration procedures is the reliance upon simple, idealized standards, for which the scattering response is assumed as theoretically postulated. The various proposed procedures differ, however, in the number, specific nature, and complexity of the used standards and in the types of measurements to be performed upon them.

Among the conflicting requirements to be satisfied by any possible standard, one which has been particularly neglected is the physical possibility of direct substitution to the unknown network, at the same interfaces where this is to be characterized.

The basic assumption of all proposed methods is the assumed independence and invariance of the error-model configuration and parameter values upon the nature and response of the unknown network to be measured. It is generally accepted to be true as long as the measurement system is time-invariant during calibration and actual measurements.

All error models and calibration procedures proposed to date consider either one-port or two-port measurements. It is generally assumed that n-port networks may be characterized by repetitive reduced measurements, performed with all but two ports closed upon "known" terminations. This is, however, a time-consuming proposition and at least one three-port network analyzer has been built and used to characterize transistor chips. More ports may be needed for characterizing microwave IC's and supercomponents.

The most common single-port error model, used in reflection measurements, is a virtual error-two-port, assumed to be inserted between the single-measurement port of an ideal error-free reflectometer and the unknown reflection to be measured. This model requires the specification of three independent complex parameters at each frequency for a complete description of its effects on reflection measurements. These parameters are frequently identified with the entries S_{11},S_{22} of the main diagonal and the product $S_{12}S_{21}$ of the other two entries of the 2×2 error-two-port scattering matrix. It is known that at least three physical-reflection standards and three calibration measurements are required to determine these parameters. Occasionally, however, more than three calibration measurements are performed to overcome an expected uncertainty of a specific standard (sliding termination) or to introduce redundancy (circle fitting). A fairly common two-port error model is the simple mirror duplication of the just-mentioned one-port model, including error-two-ports on

Manuscript received May 17, 1977.
The author is with the TRW Defense and Space Systems Group, One Space Park, Redondo Beach, CA 90278.

the outer sides of the two measurement-port interfaces where the two-port unknown networks are connected for measurement. This model is defined, of course, by six independent complex parameters, frequently identified as the entries S_{11},S_{22} and the products $S_{12}S_{21}$ of the two error-two-ports. This model requires, therefore, the measurement of at least six complex quantities at each frequency for full specification. Many of the proposed two-port calibration procedures, using this two-error-two-port model, prescribe, however, the use of many more standards and the acquisition of many more calibration data than the minimum strictly required. This redundancy of calibration standards and measurements has been so far introduced mainly to simplify the generally rather sophisticated mathematical manipulations needed to compute the error-network parameters from the calibration data.

These mathematical operations frequently involve the solution of sets of simultaneous, nonlinear complex equations which, with few fortunate exceptions, require, in general, lengthy numerical iterative processes. These mathematical complexities have led many authors to introduce, besides the just-mentioned redundancy of calibration standards and measurements, a variety of arbitrary assumptions upon the nature and size of the errors as expedients for simplifying the solution of the calibration equations and circumventing the need for slow numerical iterations. While these practices have possibly been successful to this extent, they have, however, introduced an unnecessary burden on the acquisition of the calibration data and restricted the capabilities of error removal in terms of error types and size. They also introduced problems of mutual consistency among redundant calibration data.

The most common arbitrary, simplifying assumptions intrinsic to many of the known calibration procedures are: 1) negligible measurement-port mismatch for at least one of the ports; 2) negligible response distortion due to the external interconnecting networks (cables, hinged arms, adaptors, transitions, and the like); and 3) negligible measurement-signal leakage, bypassing the unknown network.

The first two assumptions, widely the most common, consider at least one of the measurement-ports as having close-to-nominal impedance, usually 50-Ω real, and thus limit the effectiveness of the calibration to the partial removal of only the internal system errors, up to front-panel interfaces. These assumptions also set limits upon the acceptable size of the errors to be corrected.

The recently introduced Through-Short-Delay (TSD) calibration procedure [19], applies to the previously mentioned two-error-two-port error model, but, in contrast to the previously known methods, eliminates redundancies and arbitrary assumptions, with the exception of zero leakage, while providing an explicit, noniterative solution of the calibration equations. The TSD method reduces the total number of calibration standards to three, which is the minimum number of standards required to completely specify the assumed model. All TSD standards are simple two-port devices having no moving parts, which can always be designed to physically fit in place to substitute the unknown network directly at its defining interfaces. This possibility automatically includes any eventual interconnecting network within the measurement system being calibrated. In particular, the TSD procedure does not assume negligible measurement-port mismatch nor negligible response distortion by the external RF interfacing networks. As a consequence, even rather sophisticated interfacing networks may be included in the measurement circuit if needed. Wafer and microwave IC probes are interesting examples.

The delivered explicit solution of the TSD calibration equations provides closed-form expressions of the scattering parameters of the error-two-ports, which may be directly used in an explicit parameter transformation, to correct uncalibrated measurements. This procedure has been proved capable of correcting for large internal and external repeatable system errors to within the resolution and stability of the used system hardware [21], [22]. It also makes measurements possible at nonstandard impedance levels and upon non-TEM wave modes.

Because of the original choice of its error model, however, the TSD method is unable to account for errors due to signal leakage, bypassing the unknown, nor is it applicable to measurements performed upon multiport microwave networks. Recent theoretical work [23] has extended the capabilities of the TSD method to multiport S-parameter measurements, while also accounting for the errors due to all possible signal-leakage paths, bypassing the unknown network between any of its port pairs and any pairs of measurement-system ports.

No theoretical limitation was found upon the relative amount of signal leakage that can be corrected for, although increasing system resolution is expected to be required in measurement situations affected by substantial leakage.

The basic advantages of the original TSD procedure, represented by the fast execution of the explicit calibration algorithm and the ability to handle large errors, have been retained in the new, generalized explicit Super-TSD n-port calibration algorithm.

Using rather unconventional matrix algebra operators, it has been possible to express the new generalized explicit solution in a concise symbolism, directly translatable in standard programming language.

It has also been proved that the obtained matricial solution retains its validity in the conventional cases of $n = 1$ (single-port reflectometer) and $n = 2$ (conventional two-port network analyzer). In particular, assuming zero leakage in the $n = 2$ case, the Super-TSD matricial solution becomes coincident with the scalar solution already obtained for TSD.

In conclusion, it appears that the new Super-TSD algorithm includes, as particular cases, all the reflectometer and network-analyzer calibration procedures proposed to date, including those attempting to account for leakage errors. A particular class of network-analyzer errors which, however widely recognized, is not yet known to be corrected by any existing calibration procedure, arises from the need for rearranging the configuration of the internal or external microwave measurement-circuits of a network analyzer in order to perform a full two-port measurement of all four scattering parameters of an unknown network. This measurement-circuit rearrangement is obtained, partially or totally, by means of coaxial microwave switches. S-parameter test sets are, accordingly, classified as "non-switching" or "switching," depending on whether the unknown must be manually disconnected and reconnected in reverse insertion for a full four-parameter measurement.

These circuit manipulations are, in particular, known to affect the measurement-port mismatch, as seen from the unknown at either interface.

Regardless of the circuit reconfiguration being partly or totally obtained by means of microwave switches, the basic assumption of a time-invariant system, common to all known calibration and error-correction procedures, is invalidated. It then becomes interesting to ascertain whether a calibration procedure that implies cycling the measurement-circuit configuration through exactly the same steps for all the used standards and for the unknowns to be measured, could generate a global error-model representation compounding all the errors due to repeatable measurement-port mismatch changes consequent to circuit reconfiguration. In a first attempt at solving this problem, the capability of the two-port TSD method to correct for these errors has been investigated theoretically and numerically. The TSD method was selected because of the invariance of the circuit reconfiguration cycle during system calibration and measurements.

Simulated calibration data have been obtained by means of a parameter transformation, providing the erroneous scattering-parameter readings that would be generated by a network analyzer affected only by repeatable port-mismatch changes. The input data required by this transformation are the true, standard S-parameters of the considered unknown (or standard) and the assumed complex measurement-port impedances of the test set in its four configurations.

As a result of this analysis and investigation, a minor modification has been introduced in the TSD calibration algorithm to allow for a peculiar "nonreciprocity inconsistency" introduced by the port-mismatch changes.

It has also been concluded that the TSD error-model, which is common to many other previously known methods, is, in general, not suited to fully represent this type of error. A method has, however, been developed for postprocessing the scattering-parameter data obtained after the stripping of the error-two-ports from the uncalibrated measurements and removing residual switching errors not included in the model.

A similar investigation is being undertaken for the new multiport Super-TSD method in consideration of the fact that switching is bound to become mandatory if multiport measurements are to attain any acceptable degree of practicality.

An important consideration is that modern solid-state microwave switches may be expected to provide the required repeatability of response to much tighter tolerances than present-day electromechanical models and that repeatability is all that is needed if the method used does not assume any limitations upon this response.

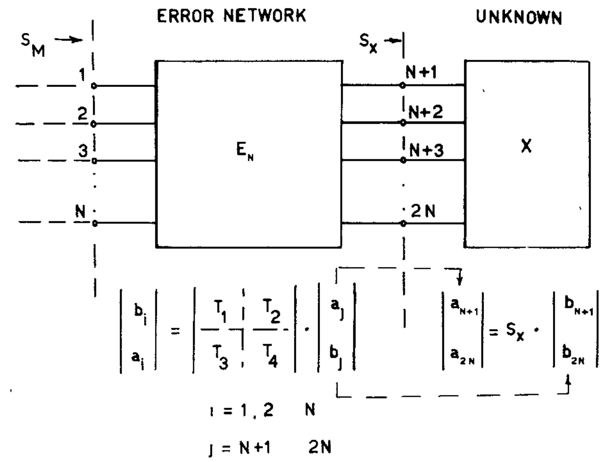

Fig. 1. The Super-TSD error model is a $2n$-port virtual error network interfacing the unknown n-port to an ideal n-port network analyzer.

II. Super-TSD

A. The n-Port Error Model

As the original TSD method, Super-TSD relies on a global representation of all system errors by means of the scattering response of a virtual, linear error-network interfacing the device under test to an ideal error-free multiport network-analyzer system. The Super-TSD error model, represented in Fig. 1, consists of a single $2n$-port E_N, embedding the unknown n-port network X. The virtual error network E_N has the n-ports $1, 2, \cdots, n$ connected to the ideal multiport network analyzer system and the n-ports $n+1, n+2, \cdots, 2n$ connected to the unknown network. These two groups of ports define the error-network input interface, where multiport scattering-parameter measurements are performed, and the inaccessible error-network output interface, where the true scattering matrix S_X of the unknown is defined.

As no assumption is made upon the behavior of the error network E_N, aside from linearity, a maximum of $(2n)^2$ independent complex parameters are required at each frequency to quantitatively describe it in matrix form.

An ideal ordered correspondence may be assumed between the ports $1, 2, \cdots, n$ of the input interface and those $n+1, n+2, \cdots, 2n$ of the output interface. Under this assumption, any of the n signal paths between an input port m and the corresponding output port $m+n$ may be considered a "direct" path, while any other path may be considered to represent signal leakage.

It is easy to see that there are $2n(n-1)$ leakage paths out of a total of $n(2n-1)$. As a consequence, $4n(n-1)$ of the $(2n)^2$ independent complex parameters, describing the error network at any frequency, represent signal leakage.

In zero-leakage situations only direct-signal paths will exist and the $2n$-port error network E_N may be sliced horizontally in n two-ports, directly connecting corresponding input and output ports. In this case, any matrix representation of E_N will contain only $4n$ nonzero entries and $4n(n-1)$ zero's.

B. The n-Port Calibration Standards

In the new Super-TSD calibration method no specific assumption is made *a priori* upon the configuration and nature of the calibration standards.

Three basic requirements are, however, assumed to be satisfied by any of the used standards.

a) All standards must be electrically connectable to physically replace the unknown X at its definition interface (ports $n+1, n+2, \cdots, 2n$), using the same type of electrical connection.

b) The scattering matrix S_{Si} of standard number i must be theoretically postulated or otherwise known from a primary measurement.

c) At least one of the standards must contain a fully known impedance-reference component.

Requirement a) implies that all the used standards be n ports as the unknown network X to be measured. Requirement b) restricts the types of components usable in standards to a few extremely simple circuit elements for which the relevant electrical parameters may be theoretically postulated or determined by primary two-port measurements. Typical elements are short circuits, open circuits, segments of beadless coaxial air line or waveguide (Throughs and Delays) and, to a lesser extent, calibrated resistors.

It is believed that a large variety of n-port calibration standards may be obtained by using different combinations of Throughs, Shorts, and Delays connected in various topological configurations among the n ports of the various standards. Also the same physical object may be reused as m different calibration standards if it is physically connectable in m of the $n!$ possible ways, while being every time defined by a different $n \times n$ scattering matrix S_{S_i}.

C. The Basic Super-TSD Calibration Equations

Following the philosophy of the Super-TSD method, system calibration is obtained by collecting at each measurement frequency the full $n \times n$ complex matrix of erroneous S-parameter readings S_{Mi}, while having standard number i with known S-matrix S_{Si} electrically substituted for the unknown X.

The most convenient representation of the error-network scattering response is given by its complex $2n \times 2n$ T-parameter matrix (Fig. 1). It can be proved (see Appendix I) that the relation between the erroneous measured scattering matrix S_{Mi}, defined at the error-network input interface, and the corresponding S_{Si} matrix is given by

$$S_{M_i} = (T_1 \cdot S_{Si} + T_2) \cdot (T_3 \cdot S_{S_i} + T_4)^{-1}, \qquad i = 1, 2, \cdots, k \tag{1}$$

where $T_1, \cdots, T_4$ are the four $n \times n$ quadrants of the $2n \times 2n$ T-matrix T. This is a matricial bilinear transformation of the postulated $n \times n$ matrix S_{Si} into the $n \times n$ measured matrix S_{Mi}. This transformation describes the n-port reflection at the input of the standard as seen through the $2n$-port linear embedding.

For $n = 1$, the error network E_N reduces to a simple two-port, the matrices S_{Mi} and S_{Si} become complex scalars with the physical meaning of reflection coefficients, and the T-matrix T becomes 2×2 complex.

It is easy to recognize that (1) then reduces to the well-known scalar bilinear transformation of the reflection of a given load impedance as seen through a linear two-port.

The Super-TSD calibration problem consists of computing the entries of the quadrants $T_1, \cdots, T_4$ of the T-matrix T from a sufficient number k of matrix pairs S_{Mi}, S_{Si}.

D. The Explicit Matricial Solution

It is easy to show that any of the k $n \times n$ matricial calibration equations (1) may be rewritten as

$$T_1 \cdot S_{Si} + T_2 - S_{Mi} \cdot T_3 \cdot S_{Si} - S_{Mi} \cdot T_4 = |0|, \qquad i = 1, 2, \cdots, k. \tag{2}$$

Each of these equations could be developed to a set of n^2 linear homogeneous equations in the entries of the error-network T-matrix. As there are $(2n)^2$ elements in the matrix, no more than four sets and four n-port standards are strictly required to solve the problem.

This type of solution is, however, not very attractive, as the formal development of the $n \times n$ matrix in the left-hand member of (2), for each standard, is already a rather elaborate operation for $n = 2$ and would become impractically complicated for n larger than 2. Besides, this type of solution would lack generality.

It is therefore interesting to investigate the possibility of solving the set (2) in matricial form by expressing the quadrants $T_1, \cdots, T_4$ as matricial functions of the matrices S_{Mi} and S_{Si}.

In these respects, it is interesting to notice that the set of k matricial equations (2) looks like a linear set of $n \times n$ homogeneous equations in the four unknown quadrants $T_1, \cdots, T_4$.

Attempts at solving this set by applying the known methods for the solution of sets of scalar linear equations would fail, however, because of the noncommutativity of matrix products and the appearance of the quadrant T_3 in the "sandwich" matrix product $S_{Mi} \cdot T_3 \cdot S_{Si}$.

It is, however, possible to break the sandwich product by applying a transformation that uses two matrix operators known in multilinear algebra.

These are the Kroneker tensor product $A \otimes B$ of two matrices A and B [24, pp. 235–236] and the stacking operator $S(A)$ of a matrix A [24, p. 245, problem 16]. Applied to $n \times n$ matrices A and B, the Kroneker product generates an $n^2 \times n^2$ product matrix, while the stacking operator transforms an $n \times n$ matrix A into the n^2-dimensional column vector $S(A)$ by sequentially stacking the columns of A in vertical order.

Alternatively, a "reshuffle" or "row-stacking" operator $RS(A) = S(A^T)$ may be used which is equal to the stacking of the transpose A^T of the matrix A. $RS(A)$ is therefore an

n^2-dimensional column vector, sequentially containing, in vertical order, the rows of A rotated 90° clockwise.

The breaking of the sandwich matrix product $A \cdot C \cdot B$ may then be obtained in either of the two following forms:

$$S(A \cdot C \cdot B) = (B^T \otimes A) \cdot S(C)$$

or

$$RS(A \cdot C \cdot B) = (A \otimes B^T) \cdot RS(C).$$

Using the second form and completing with the $n \times n$ unit matrix I the nonsandwich products of (2), this may be rewritten as

$$(I \otimes S_{Si}^T) \cdot RS(T_1) + RS(T_2) - (S_{Mi} \otimes S_{Si}^T) \cdot RS(T_3) - (S_{Mi} \otimes I) \cdot RS(T_4) = |0| \quad (2')$$

where $i = 1, 2, \cdots, k$.

The advantage of (2') is that all its terms are matrix-by-vector products of the same order n^2, and thus these equations form a set of linear, homogeneous matricial equations in the four column vectors $RS(T_i)$, $i = 1, 2, \cdots, 4$.

Provided at least three n-port standards are measured during system calibration ($k = 3$), the following explicit vectorial solution may be obtained from the set (2') by Gaussian elimination (see Appendix II):

$RS(T_1) =$ Arbitrary, nonzero complex column vector of order n^2 (3)

$$RS(T_2) = \{(S_{M1} \otimes S_{S1}^T)(B^{-1}A - D^{-1}C)^{-1}(B^{-1}E - D^{-1}F) + (S_{M1} \otimes I)(A^{-1}B - C^{-1}D)^{-1}(A^{-1}E - C^{-1}F) - (I \otimes S_{S1}^T)\} \cdot RS(T_1) \quad (4)$$

$$RS(T_3) = (B^{-1}A - D^{-1}C)^{-1} \cdot (B^{-1}E - D^{-1}F) \cdot RS(T_1) \quad (5)$$

$$RS(T_4) = (A^{-1}B - C^{-1}D)^{-1} \cdot (A^{-1}E - C^{-1}F) \cdot RS(T_1) \quad (6)$$

where the auxiliary $n^2 \times n^2$ matrices $A, B, \cdots, E, F$ are defined by

$$A = (S_{M1} \otimes S_{S1}^T) - (S_{M2} \otimes S_{S2}^T) \quad (7)$$

$$B = (S_{M1} \otimes I) - (S_{M2} \otimes I) \quad (8)$$

$$C = (S_{M1} \otimes S_{S1}^T) - (S_{M3} \otimes S_{S3}^T) \quad (9)$$

$$D = (S_{M1} \otimes I) - (S_{M3} \otimes I) \quad (10)$$

$$E = (I \otimes S_{S1}^T) - (I \otimes S_{S2}^T) \quad (11)$$

$$F = (I \otimes S_{S1}^T) - (I \otimes S_{S3}^T). \quad (12)$$

As indicated by (3)–(6), it would appear that this explicit matricial solution implies some degree of arbitrariness for the quadrant T_1 of the error-network T-matrix. Besides, the choice of T_1 as an independent matricial variable is also arbitrary, as the Gaussian elimination process could be conducted in such a way as to leave any of the four quadrants arbitrary, while expressing the other three as matricial functions thereof.

This is, in a way, consistent with the homogeneous character of the set (2). The existence of restrictions to the completely arbitrary choice of the independent quadrant is, however, suspected and expected. First, it is quite evident that it would be meaningless to choose a zero matrix. Second, it would not be useful to choose the quadrant such that the total T-matrix T of the error network is singular (see Section II-D). Also, within the limits of the arbitrariness, (1) should be invariant to the choice. In these respects, if T_1 is the quadrant chosen to be independent and it commutes with all the S_{Si}, then

$$S_{Mi} = (T_1 S_{Si} + T_2) \cdot (T_3 S_{Si} + T_4)^{-1} = (S_{Si} + T_2 T_1^{-1}) \cdot (T_3 T_1^{-1} S_{Si} + T_4 T_1^{-1})^{-1}$$

so that only the three matrices $T_2 T_1^{-1}$, $T_3 T_1^{-1}$, and $T_4 T_1^{-1}$ would be required to specify the matricial bilinear transformation (1).

E. The n-Port Deembedment Formula

Once the $2n \times 2n$ T-matrix T of the error network has been computed and assuming that it is nonsingular, the removal of all system calibration errors from the measured $n \times n$ scattering parameter matrix S_M of an unknown n-port X (Fig. 1) may be performed by computing the "corrected" $n \times n$ scattering-parameter matrix S_X as (see Appendix III):

$$S_X = (R_1 \cdot S_M + R_2)(R_3 \cdot S_M + R_4)^{-1} = \{(T_1 - T_2 T_4^{-1} T_3)^{-1} \cdot S_M + (T_3 - T_4 T_2^{-1} T_1)^{-1}\} \cdot \{(T_2 - T_1 T_3^{-1} T_4)^{-1} \cdot S_M + (T_4 - T_3 T_1^{-1} T_2)^{-1}\}^{-1} \quad (13)$$

where $R_1, \cdots, R_4$ are the quadrants of the inverse T^{-1} of the T-matrix T of the error network E_N.

The second form of the right-hand member of (13) is only usable if the quadrants of T are individually nonsingular, a condition that may not be satisfied even if the total matrix T is nonsingular. If the T_i are nonsingular, however, four $n \times n$ matrices must be inverted instead of a big $2n \times 2n$ matrix, a circumstance that may be advantageous to numerical inversion in relation to rounding off errors. Also the second form of (13) greatly simplifies if the quadrants $T_1, \cdots, T_4$ of T mutually commute, in which case

$$S_X = (T_4 \cdot S_M - T_2)(-T_3 \cdot S_M + T_1)^{-1}. \quad (13')$$

Being equivalent to (1), the n-port deembedment formula (13) should be invariant to the choice of the quadrant T_1 under the same arbitrariness restrictions.

F. Special Cases

It is interesting to consider the two Super-TSD special cases for $n = 1$ and $n = 2$ with zero leakage.

For $n = 1$, which is the classical case of the single-port microwave reflectometer, the error network E_N reduces to a simple two-port with T-matrix: $T_1 = T_{11}$, $T_2 = T_{12}$, $T_3 = T_{21}$, and $T_4 = T_{22}$.

The measured and postulated scattering matrices S_{Mi} and S_{Si} become complex scalar reflection coefficients

$$S_{Mi} = \Gamma_{mi} \qquad S_{Si} = \Gamma_{si}, \qquad i = 1, 2, 3$$

and the Kroneker products reduce to simple scalar products

$$S_{Mi} \otimes S_{Si}^T = \Gamma_{mi} \cdot \Gamma_{si} \qquad S_{Mi} \otimes I = \Gamma_{mi} \qquad I \otimes S_{Si}^T = \Gamma_{si}.$$

Equation (1) then becomes the well-known scalar bilinear transformation of the single-port microwave reflectometer [15, p. 399, eq. (1)]

$$\Gamma_{mi} = \frac{T_{11} \cdot \Gamma_{si} + T_{12}}{T_{21} \cdot \Gamma_{si} + T_{22}} = \frac{S_{11} - \text{DET}(S) \cdot \Gamma_{si}}{1 - S_{22} \cdot \Gamma_{si}}, \quad i = 1, 2, 3 \quad (1')$$

where S_{11}, S_{22}, and $\text{DET}(S) = S_{11}S_{22} - S_{12}S_{21}$ are S-parameters and the determinant of the scattering matrix of the error two-port.

It is easy to show that, while the auxiliary matrices $A, \cdots, F$ become simple complex scalars, the Super-TSD solution (3)–(6) provides the well-known microwave reflectometer calibration procedure relying on three reflection standards

$$RS(T_2) = T_{12} = -\frac{\Gamma_{m1}\Gamma_{m2}\Gamma_{s3}(\Gamma_{s1} - \Gamma_{s2}) - \Gamma_{m1}\Gamma_{m3}\Gamma_{s2}(\Gamma_{s1} - \Gamma_{s3}) + \Gamma_{m2}\Gamma_{m3}\Gamma_{s1}(\Gamma_{s2} - \Gamma_{s3})}{(\Gamma_{m1} - \Gamma_{m3})(\Gamma_{m1}\Gamma_{s1} - \Gamma_{m2}\Gamma_{s2}) - (\Gamma_{m1} - \Gamma_{m2})(\Gamma_{m1}\Gamma_{s1} - \Gamma_{m3}\Gamma_{s3})} T_{11} \quad (14)$$

$$RS(T_3) = T_{21} = \frac{(\Gamma_{m1} - \Gamma_{m3})(\Gamma_{s1} - \Gamma_{s2}) - (\Gamma_{m1} - \Gamma_{m2})(\Gamma_{s1} - \Gamma_{s3})}{(\Gamma_{m1} - \Gamma_{m3})(\Gamma_{m1}\Gamma_{s1} - \Gamma_{m2}\Gamma_{s2}) - (\Gamma_{m1} - \Gamma_{m2})(\Gamma_{m1}\Gamma_{s1} - \Gamma_{m3}\Gamma_{s3})} T_{11} \quad (15)$$

$$RS(T_4) = T_{22} = \frac{(\Gamma_{s1} - \Gamma_{s3})(\Gamma_{m1}\Gamma_{s1} - \Gamma_{m2}\Gamma_{s2}) - (\Gamma_{s1} - \Gamma_{s2})(\Gamma_{m1}\Gamma_{s1} - \Gamma_{m3}\Gamma_{s3})}{(\Gamma_{m1} - \Gamma_{m3})(\Gamma_{m1}\Gamma_{s1} - \Gamma_{m2}\Gamma_{s2}) - (\Gamma_{m1} - \Gamma_{m2})(\Gamma_{m1}\Gamma_{s1} - \Gamma_{m3}\Gamma_{s3})} T_{11}. \quad (16)$$

For $n = 2$ and zero leakage, the error network E_N, sliced horizontally between the two pairs of corresponding ports, reduces to the TSD two-error-two-port model. It is then easy to prove that the quadrants $T_1, \cdots, T_4$ of the T-matrix T may be related to the T- and S-matrices of the two-error two-ports as follows:

$$T_1 = \left|\begin{array}{c|c} T_{11A} & 0 \\ \hline 0 & T_{11B}/\text{DET}(T_B) \end{array}\right| = \left|\begin{array}{c|c} -\text{DET}(S_A)/S_{21A} & 0 \\ \hline 0 & -\text{DET}(S_B)/S_{12B} \end{array}\right| \quad (17)$$

$$T_2 = \left|\begin{array}{c|c} T_{12A} & 0 \\ \hline 0 & -T_{21B}/\text{DET}(T_B) \end{array}\right| = \left|\begin{array}{c|c} S_{11A}/S_{21A} & 0 \\ \hline 0 & S_{22B}/S_{12B} \end{array}\right| \quad (18)$$

$$T_3 = \left|\begin{array}{c|c} T_{21A} & 0 \\ \hline 0 & -T_{12B}/\text{DET}(T_B) \end{array}\right| = \left|\begin{array}{c|c} -S_{22A}/S_{21A} & 0 \\ \hline 0 & -S_{11B}/S_{12B} \end{array}\right| \quad (19)$$

$$T_4 = \left|\begin{array}{c|c} T_{22A} & 0 \\ \hline 0 & T_{22B}/\text{DET}(T_B) \end{array}\right| = \left|\begin{array}{c|c} 1/S_{21A} & 0 \\ \hline 0 & 1/S_{12B} \end{array}\right| \quad (20)$$

where the parameters with indices ijA belong to the error-two-port A, and those with indices ijB belong to the error-two-port B. Also

$$\text{DET}(T_A) = S_{12A}/S_{21A} \qquad \text{DET}(T_B) = S_{12B}/S_{21B}$$

are the determinants of the respective T-matrices.

On the basis of these expressions, (1), written for the unknown network X as in (1.11) of Appendix I, becomes equivalent to the TSD embedding equations

$$S_{11M} = \frac{S_{11A} - S_{11X}\,\text{DET}(S_A) - S_{11B}[S_{11A}S_{22X} - \text{DET}(S_A)\,\text{DET}(S_X)]}{1 - S_{11X}S_{22A} - S_{11B}[S_{22X} - S_{22A}\,\text{DET}(S_X)]} \quad (21)$$

$$S_{12M} = \frac{S_{12A} \cdot S_{12X} \cdot S_{12B}}{1 - S_{11X}S_{22A} - S_{11B}[S_{22X} - S_{22A}\,\text{DET}(S_X)]} \quad (22)$$

$$S_{21M} = \frac{S_{21A} \cdot S_{21X} \cdot S_{21B}}{1 - S_{11X}S_{22A} - S_{11B}[S_{22X} - S_{22A}\,\text{DET}(S_X)]} \quad (23)$$

$$S_{22M} = \frac{(1 - S_{11X}S_{22A})S_{22B} - [S_{22X} - S_{22A}\,\text{DET}(S_X)]\,\text{DET}(S_B)}{1 - S_{11X}S_{22A} - S_{11B}[S_{22X} - S_{22A}\,\text{DET}(S_X)]} \quad (24)$$

which are, in turn, equivalent to the cascaded T-matrix product $T_M = T_A \cdot T_X \cdot T_B$.

At the same time, (13), which assumes the form (13′) because of the diagonal character of the quadrants $T_1, \cdots, T_4$, becomes equivalent to the TSD deembedment equations

$$S_{11X} = \frac{S_{11B}[S_{11A}S_{22M} - \mathrm{DET}\,(S_M)] + (S_{11M} - S_{11A})\,\mathrm{DET}\,(S_B)}{S_{11B}[S_{22M}\,\mathrm{DET}\,(S_A) - S_{22A}\,\mathrm{DET}\,(S_M)] + [S_{11M}S_{22A} - \mathrm{DET}\,(S_A)]\,\mathrm{DET}\,(S_B)} \tag{25}$$

$$S_{12X} = \frac{-S_{12M} \cdot S_{21A} \cdot S_{21B}}{S_{11B}[S_{22M}\,\mathrm{DET}\,(S_A) - S_{22A}\,\mathrm{DET}\,(S_M)] + [S_{11M}S_{22A} - \mathrm{DET}\,(S_A)]\,\mathrm{DET}\,(S_B)} \tag{26}$$

$$S_{21X} = \frac{-S_{21M} \cdot S_{12A} \cdot S_{12B}}{S_{11B}[S_{22M}\,\mathrm{DET}\,(S_A) - S_{22A}\,\mathrm{DET}\,(S_M)] + [S_{11M}S_{22A} - \mathrm{DET}\,(S_A)]\,\mathrm{DET}\,(S_B)} \tag{27}$$

$$S_{22X} = \frac{S_{22M}\,\mathrm{DET}\,(S_A) - S_{22A}\,\mathrm{DET}\,(S_M) + [S_{11M}S_{22A} - \mathrm{DET}\,(S_A)]S_{22B}}{S_{11B}[S_{22M}\,\mathrm{DET}\,(S_A) - S_{22A}\,\mathrm{DET}\,(S_M)] + [S_{11M}S_{22A} - \mathrm{DET}\,(S_A)]\,\mathrm{DET}\,(S_B)} \tag{28}$$

which are, in turn, equivalent to the cascaded T-matrix product $T_X = T_A^{-1} \cdot T_M \cdot T_B^{-1}$ [19, p. 72, eq. (19)].

The element of arbitrary choice in the quadrant T_1, as expressed by (17), is introduced by the possibility of multiplying S_{21A} and S_{12B} by an arbitrary complex scalar, provided S_{12A} and S_{21B} are at the same time divided by the same scalar. This simultaneous scaling of the S_{ij} $(i \neq j)$ maintains the values of the products $S_{12A}S_{21A}$ and $S_{12B}S_{21B}$ unchanged (see Appendix V) and does not invalidate the obtained error two-port solution as a representation of the relationship between the measured and the true parameter matrices S_M and S_X.

It is interesting to notice that the diagonal character of the matrix quadrants $T_1, \cdots, T_4$ is a consequence of the zero-leakage assumption. In the absence of leakage, the quadrants of the T-matrix T are always diagonal matrices for any number of ports $n > 1$. As a consequence they mutually commute and the n-port deembedment formula (13) always reduces to the much simpler form (13′).

A formal development of the Super-TSD solution (3)–(6) for $n = 2$ and nonzero leakage is being worked out at this writing, in order to confirm as a special case the explicit TSD algorithm and study by inspection the sensitivities of the Super-TSD solution to tolerances upon the standards and numerical rounding off errors. In conclusion, it appears that by appropriate choice of the specific standards used, the Super-TSD calibration procedure may be shown to include, as particular cases, all the known microwave reflectometer and two-port network-analyzer calibration procedures; in particular, all those attempting to correct for leakage errors by inclusion of one or two leakage paths in the error model.

III. Switching Errors

A. *Types and Origins of Switching Errors*

It is widely recognized that the rearrangement of the microwave measurement-circuit configuration required for a full two-port measurement introduces two classes of measurement errors.

First, regardless of the circuit rearrangement being obtained manually or by means of microwave switches, the repetitive making and breaking of contacts introduces an erratic component in the circuit response for all four of the required situations. Electromechanical microwave switches have traditionally been considered with much reservation in these respects.

This class of errors obviously cannot be corrected by any calibration procedure, due to their erratic statistical nature. There is hope, however, that technological progress, in particular the development of modern solid-state switches, may reduce the proportions of this nonrepeatability of response.

Even taking this for granted, however, another class of repeatable, systematic errors will remain, which is directly related to the cyclic reconfiguration of the measurement circuit.

These errors arise first from the practical impossibility of designing a test set to have measurement ports of perfectly nominal impedance at every frequency, and, second, from the dependence of the measurement-port mismatch, at any given frequency, upon the specific measurement-circuit configuration being used.

Practical systems will thus always be affected by varying degrees of port mismatch at different frequencies and, on top of that, the port mismatch at any given frequency will change cyclically following the circuit-configuration switching. In the light of this conclusion, the need for external calibration standards, including at least one traceable impedance-reference component, becomes mandatory. The use of segments of beadless coaxial air line or waveguide for this purpose is gaining widespread consensus.

In a completely general situation, the two measurement ports of a switching test set will assume four pairs of uncorrelated complex impedance values during a complete two-port measurement cycle. It is thus logical to assume that, in the absence of any other type of error, the test set readings would correspond to the scattering parameters of the unknown network X, normalized to the specific complex port impedances the set has during the measurement of each individual parameter.

Although the concept of normalization with respect to complex port impedances is known [25], [26], an unusual situation arises here, because of each scattering matrix element being normalized to a different pair of complex port impedances. We believe such a scattering matrix deserves the qualification "supergeneralized."

TABLE I
LABELING OF MEASUREMENT—PORT IMPEDANCES IN A SWITCHING SCATTERING-PARAMETER TEST SET

Measurement	Impedance Port 1	Impedance Port 2
S_{11}	Z_{111}	Z_{211}
S_{12}	Z_{112}	Z_{212}
S_{21}	Z_{121}	Z_{221}
S_{22}	Z_{122}	Z_{222}

B. Simulation of Repeatable Mismatch Changes

The theoretical simulation and modeling of the repeatable switching errors due to consistently cyclic measurement-port mismatch changes is the only reliable basis for a study of the properties of these errors and for an accurate analysis of the capability of any calibration procedure to correct them.

A numerical error simulation also establishes a quantitative correlation between the errors and the port mismatches that cause them. It would be impossible to accomplish all this experimentally. The fundamental tool of such a simulation must be a parameter transformation providing the supergeneralized scattering-parameter matrix S_X^* of a given network X as function of the true (postulated) standard scattering matrix S_X (normalized to 50-Ω real) and of the nonnominal complex port impedances the test set is assumed to have during the various steps of a measurement cycle.

A two-port S to S^* transformation has been obtained (see Appendix IV) which contains as parameters the four pairs of complex port impedances Z_{nij} of the test set (n = port number, ij = indices of the scattering parameter being measured. See Table I). This S to S^* transformation is given by

$$S_{11}^* = \frac{C_{111}X + C_{211}Y + C_{311}U + C_{411}V}{C_{211}X + C_{111}Y + C_{411}U + C_{311}V} \tag{29}$$

$$S_{12}^* = \frac{4S_{12}}{C_{212}X + C_{112}Y + C_{412}U + C_{312}V} \tag{30}$$

$$S_{21}^* = \frac{4S_{21}}{C_{221}X + C_{121}Y + C_{421}U + C_{321}V} \tag{31}$$

$$S_{22}^* = -\frac{C_{122}X + C_{222}Y - C_{322}U - C_{422}V}{C_{222}X + C_{122}Y + C_{422}U + C_{322}V} \tag{32}$$

where

$$X = 1 - \text{DET}(S) \tag{33}$$

$$U = 1 + \text{DET}(S) \tag{34}$$

$$Y = S_{11} - S_{22} \tag{35}$$

$$V = S_{11} + S_{22} \tag{36}$$

with

$$\text{DET}(S) = S_{11}S_{22} - S_{12}S_{21} \tag{37}$$

while the 16 coefficients C_{mij} are expressed by

$$C_{1ij} = \sqrt{\frac{Z_{2ij}}{Z_{1ij}}} - \sqrt{\frac{Z_{1ij}}{Z_{2ij}}} \tag{38}$$

$$C_{2ij} = \sqrt{\frac{Z_{2ij}}{Z_{1ij}}} + \sqrt{\frac{Z_{1ij}}{Z_{2ij}}} \tag{39}$$

$$C_{3ij} = \frac{Z_0}{\sqrt{Z_{1ij}Z_{2ij}}} - \frac{\sqrt{Z_{1ij}Z_{2ij}}}{Z_0} \tag{40}$$

$$C_{4ij} = \frac{Z_0}{\sqrt{Z_{1ij}Z_{2ij}}} + \frac{\sqrt{Z_{1ij}Z_{2ij}}}{Z_0} \tag{41}$$

$$C_{2ij}^2 - C_{1ij}^2 = C_{4ij}^2 - C_{3ij}^2 = 4. \tag{42}$$

In this transformation, the a_i and b_i waves at the two ports are assumed to be the traveling waves, as defined by Kurokawa in [26, p. 201, eq. (43)]. This assumption is motivated by the fact that it is not clear to us how a network analyzer could be sensitive to Youla's power waves [25]. In any case, even assuming the system to be sensitive to power waves, the form of the transformation (29)–(32) would be the same, only the definition of the C_{mij} coefficients would change (see Appendix IV).

C. Properties of the Repeatable Switching Errors

A number of characteristic properties of these port-mismatch switching errors may be predicted by inspection of the given S to S^* transformation. First, it is easy to see that a matched two-port will, in general, appear to be mismatched, unless by coincidence $C_{111} = C_{311} = C_{122} = C_{322} = 0$. Second, a reciprocal two-port with the ratio $S_{12}/S_{21} = 1$ will appear to be nonreciprocal because of a ratio $S_{12}^*/S_{21}^* \neq 1$, which, in turn, depends on both the port impedances Z_{nij} and the intrinsic response of the two port itself, as characterized by its standard S-parameters. In particular, different lengths of transmission line with nominal impedance will show different amounts of apparent nonreciprocity, as measured by the apparent nonreciprocity ratio S_{12}^*/S_{21}^*.

Also, all these properties are quantitatively dependent upon the direction of insertion of the measured two-port with respect to the system's measurement ports, being either "forward" (Port 1 to Port 1, Port 2 to Port 2) or "reverse" (Port 1 to Port 2 and Port 2 to Port 1). This is true in the sense of comparing the forward parameters to those obtained from the reverse measurement after exchanging the diagonally opposite elements (S_{11} to S_{22} and S_{12} to S_{21}).

This is a consequence of the fact that, in general, the system is "polarized" or "asymmetric" with respect to the nonnominal complex port-impedances. Any two-port will

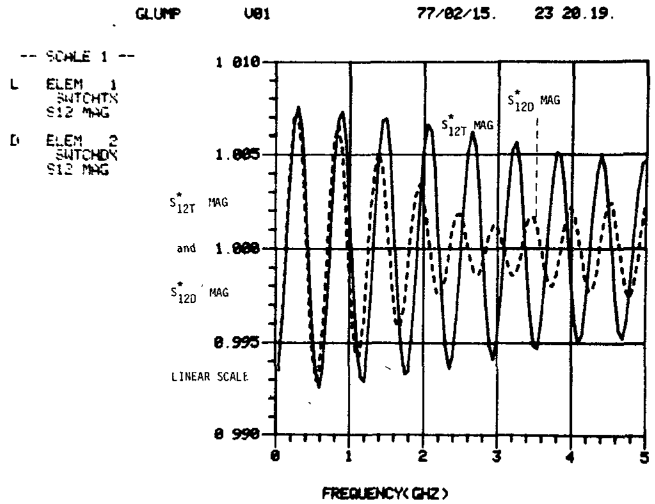

Fig. 2. Apparent backward transmission of the Through and Delay in the presence of repeatable switching errors due to frequency-dependent port mismatch.

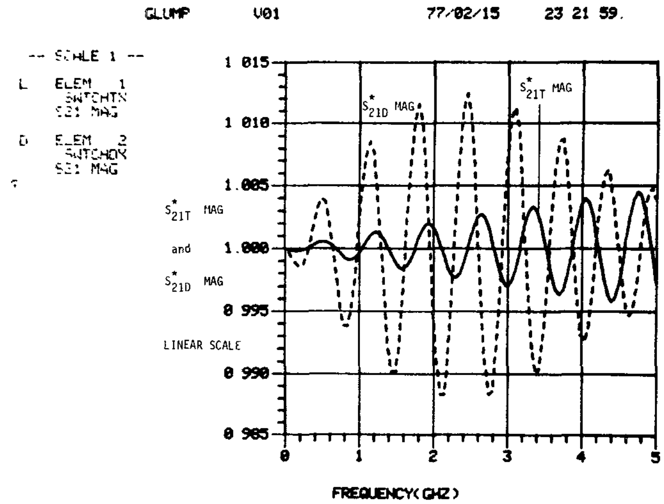

Fig. 3. Apparent forward transmission of the Through and Delay in the presence of switching errors. Comparison with Fig. 2 shows different apparent nonreciprocity ratios S_{12}/S_{21}.

thus have two measured S^*-matrices: S_F^* measured in forward insertion and S_B^* measured in backward insertion, with subsequent exchange of the diagonally opposite elements.

D. The Behavior of TSD Under Switching Errors

Simulated TSD calibration data may be obtained from the S to S^* transformation (29)–(32) by assuming the port impedances Z_{nij} to be known functions of the frequency and by substituting, as standard S-parameters, numerical values of the elements of the S_{ST}, S_{SS}, and S_{SD} matrices defined by equations (5.1)–(5.3) of Appendix V.

In this way, simulated S_T, S_S, and S_D matrices (see Appendix V) are obtained that may be used as test input data for the TSD algorithm. At the same time, the standard S-parameters S_X of a known network X may be used to compute its S_X^*-parameters under the same Z_{nij} assumptions. The obtained S_X^* matrix then represents a simulated uncalibrated measurement affected by the same switching errors as the S_T, S_S, and S_D data.

The computed scattering matrices S_A and S_B of the error two-ports A and B, obtained from TSD, may then be stripped, with (25)–(28), from the S_X^*-matrix, and the resulting matrix S_{SX} ("stripped" S_X) compared to the original S_X-matrix, used as input to the S to S^* transformation.

Such a numerical simulation of a network analyzer, affected only by port-mismatch switching errors, has already been performed by assuming the impedances Z_{nij} to be equivalent to eight different resistive loads R_{nij} located at eight different electrical distances θ_{nij} from the external measurement-port interfaces, deep inside the switching test set, according to the expressions

$$Z_{nij} = \frac{r_{nij} + j \tan \theta_{nij}}{1 + jr_{nij} \tan \theta_{nij}} Z_0, \qquad \left(r_{nij} = \frac{R_{nij}}{Z_0}\right). \tag{43}$$

The specific values of the real loads R_{nij} and of their distances θ_{nij} from the port interfaces have been changed randomly in switching from one Z_{nij} to the other, thus introducing 16 arbitrary parameters in the port impedances. These, as a consequence, change with frequency in mutually uncorrelated ways.

Typical examples of the simulated calibration data obtained for the Through and the Delay are shown in Figs. 2 and 3. It is easy to see that the S to S^* transformation leaves the data for the Short unchanged, so that $S_S = S_{SS}$ [Appendix V, (5.2) and (5.5)].

An analysis of the reduced expressions of S_{12}^* and S_{21}^* for the Through and the Delay, obtained from (30) and (31), shows a mutual inconsistency of these data that cannot be accounted for in the simple TSD error model. This inconsistency is due to the apparent nonreciprocity ratios of the Through and of the Delay, expressed by

$$\frac{S_{12}^*}{S_{21}^*} = \frac{(C_{221} + C_{421})e^{\rho_i} + (C_{221} - C_{421})e^{-\rho_i}}{(C_{212} + C_{412})e^{\rho_i} + (C_{212} - C_{412})e^{-\rho_i}} \neq 1, \qquad i = 1, 2 \tag{44}$$

being, in general, different in the presence of switching errors, while, according to the error model, it should be

$$\frac{S_{12T}}{S_{21T}} = \frac{S_{12D}}{S_{21D}} = \frac{S_{12A}}{S_{21A}} \frac{S_{12B}}{S_{21B}}. \tag{45}$$

This inconsistency may, however, be removed by assuming the impedance-reference line L_2, used in the delay standard, to be "virtually" nonreciprocal, with

$$S_{SD} = \begin{vmatrix} 0 & e^{-\rho_2'} \\ e^{-\rho_2''} & 0 \end{vmatrix} \tag{46}$$

$$T_{SD} = \begin{vmatrix} e^{-\rho_2'} & 0 \\ 0 & e^{\rho_2''} \end{vmatrix} = e^{-\delta} \begin{vmatrix} e^{-\rho_2} & 0 \\ 0 & e^{\rho_2} \end{vmatrix} \tag{47}$$

where

$$\delta = \tfrac{1}{2}(\rho_2' - \rho_2'') \tag{48}$$

$$\rho_2 = \tfrac{1}{2}(\rho_2' + \rho_2''). \tag{49}$$

This assumption does not invalidate usage of the line L_2 as an impedance standard and does not introduce any

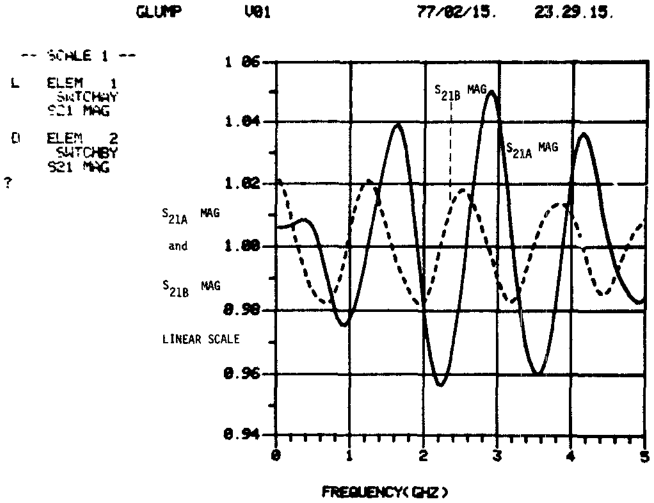

Fig. 4. Magnitude of the forward transmission for the error two-ports A and B, computed from simulated TSD calibration data representing switching errors.

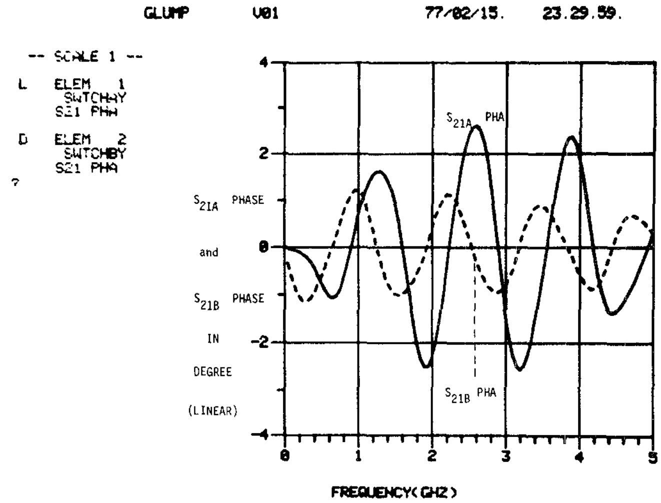

Fig. 5. Phase of the forward transmission for the error two-ports A and B computed from simulated TSD calibration data representing switching errors.

new unknowns, as the nonreciprocity is uniquely determined by the known mutual inconsistency of the through and delay data. The S-parameters of error two-ports A and B may still be explicitly computed with the same algorithm (Appendix V) if the fundamental matrix products

$$H = T_{D1} T_{D2}^{-1} \quad (50)$$

$$K = T_{D2}^{-1} T_{D1} \quad (51)$$

are redefined as

$$H_{\text{NEW}} = e^{-\delta}(T_{D1} T_{D2}^{-1}) \quad (52)$$

$$K_{\text{NEW}} = e^{-\delta}(T_{D2}^{-1} T_{D1}) \quad (53)$$

where

$$e^{-\delta} = \frac{1}{\sqrt{\text{DET }(H)}} = \frac{1}{\sqrt{\text{DET }(K)}} = \sqrt{\frac{S_{12D}}{S_{21D}} \cdot \frac{S_{21T}}{S_{12T}}}. \quad (54)$$

Following this redefinition, it is

$$\text{DET }(H_{\text{NEW}}) = \text{DET }(K_{\text{NEW}}) = 1 \quad (55)$$

instead of

$$\text{DET }(H) = \text{DET }(K) = \frac{S_{12T}}{S_{21T}} \frac{S_{21D}}{S_{12D}} = e^{2\delta} \neq 1. \quad (56)$$

Error two-port solutions S_A and S_B have been computed with a TSD error-computation program, modified according to (52) and (53). These error two-ports appear to represent at least part of the errors due to the repeatable port-impedance changes as shown in Figs. 4 and 5. Indeed, by stripping the obtained error two-ports A and B from the Short (S_S), we obtain a residual network having $S_{11} = S_{22} = -1$ and $S_{12} = S_{21} = 0$, equivalent to an immediate short at both measurement interfaces (S_{SS}). Also, by stripping the error two-ports from the Through (S_T) and Delay (S_D), residual networks are obtained having $S_{11} = S_{22} = 0$ (Fig. 6) to within the rounding off errors of the processor.

In transmission, however, both the Through- and the Delay-residual networks show residual magnitude and phase ripples around the expected smooth values of S_{12} and S_{21} of the lines L_1 and L_2, which are flat in magnitude and linear in phase.

Our present conclusion is that, although the two-port TSD model is fully capable of representing any errors other than switching, including any external interfacing networks, it cannot, in general, represent the totality of the switching errors. In practical situations, compounding switching and nonswitching errors, the stripping of the computed error two-ports A and B will remove the totality of the nonswitching errors and an unspecified part of the switching errors. This is a consequence of the fact that the S_X to S_M transformation, defined by (21)–(24), cannot completely match the S to S^* transformation (29)–(32), irrespective of both having the same total number of parameters (the Z_{nij} and the elements of S_A and S_B). This behavior, probably due to the fact that only the products $S_{12} S_{21}$ of the error two-ports are relevant, must be shared by all calibration procedures using the same error model.

It will be interesting, in these respects, to investigate whether the leakage entries of the Super-TSD error network T-matrix T may provide a full representation of the switching errors for $n = 2$ by increasing the number of available model parameters. We are assuming, however, that the residual switching errors, left after stripping the error two-ports, behave as the total switching errors and may thus be represented by an S to S^* transformation based on "equivalent" port impedances Z_{nij} and corresponding C_{mij} coefficients.

A method has been developed for computing the equivalent C_{mij} coefficients of the residual switching errors, from the scattering parameters of the "Residual-Through," of the "Residual-Delay," and those of an auxiliary reciprocal, nonsymmetric network, to be measured as a reference unknown in forward and backward insertion, during system calibration. The obtained C_{mij} coefficients are then used to post-

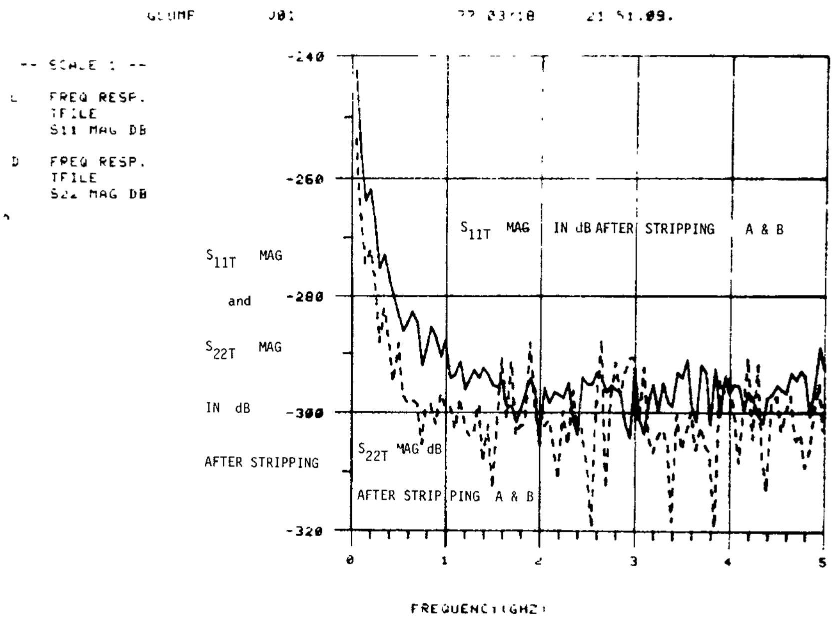

Fig. 6. The residual network obtained by stripping the error two-ports A and B from the Through data appears to be matched to within the rounding off errors of the used processor (CDC 6400).

process the S-parameter data obtained from the stripping of the error two-ports for final correction. The expressions of the equivalent C_{mij} coefficients must be omitted, here due to space limitations, but will be reported with results of the postprocessing in a future paper.

IV. Conclusion

A new calibration procedure for automated network analyzers has been developed that extends the basic philosophy of the TSD method to multiport scattering-parameter measurements affected by multiple signal leakage.

The new Super-TSD procedure computes a global error representation for the whole measurement system down to the defining of interfaces of the unknown network by means of explicit matricial expressions using the measured scattering parameters of at least three multiport standards.

Various combinations of Throughs, Shorts, and Delays may be used as n-port standards to calibrate an n-port network analyzer system.

A preliminary study has also been conducted to investigate the capability of the classical two-port TSD method to correct the switching errors due to repeatable measurement-port mismatch changes typical of switching S-parameter test sets. It appears that TSD can be made to correct for these errors too by introducing a minor mathematical modification and by adding data postprocessing after the deembedment of the error two-ports from the uncalibrated measurements.

Future research activities should aim at uncovering the restrictions to the arbitrary choice of one of the quadrants of the error-network T-matrix in Super-TSD and investigate this method in relation to the correction of repeatable switching errors. The sensitivity of the method to tolerances upon the parameters of the used standards and to the rounding off errors generated by practical processors are subjects of extreme engineering interest.

Appendix I

By definition, the T-parameter matrix of the $2n$-port error-network (Fig. 1) relates the vector of the input-interface waves b_i,a_i to the vector of the output-interface waves a_j,b_j according to the matrix expression

$$\begin{vmatrix} b_i \\ a_i \end{vmatrix} = T \cdot \begin{vmatrix} a_j \\ b_j \end{vmatrix} = \begin{vmatrix} T_1 & T_2 \\ T_3 & T_4 \end{vmatrix} \cdot \begin{vmatrix} a_j \\ b_j \end{vmatrix} \tag{1.1}$$

$$i = 1, 2, \cdots, n \tag{1.2}$$

$$j = n+1, n+2, \cdots, 2n. \tag{1.3}$$

In (1.1) waves b_i and b_j emerge from the error network at the input and output interfaces, respectively. Similarly, waves a_i and a_j are incident on and propagating towards the error network at these interfaces.

As seen from the unknown network, however (or any of the calibration standards), the roles of the output-interface waves a_j and b_j are obviously interchanged. Waves b_j emerging from the error network are incident upon the unknown (or the standard), while waves a_j, incident upon the error network, emerge from the unknown (or the standard). As a consequence, waves a_j are related to waves b_j through the $n \times n$ scattering matrix S_X of the unknown (or the matrix S_{Si} of standard number i)

$$|a_j| = S_X \cdot |b_j|$$

or

$$|a_j| = S_{Si} \cdot |b_j|. \tag{1.4}$$

Because of this relation, the vector a_j,b_j of the error-network output-interface waves, appearing in the right-hand member of (1.1), may be written as

$$\begin{vmatrix} a_j \\ b_j \end{vmatrix} = \begin{vmatrix} S_X & Z \\ Z & I \end{vmatrix} \cdot \begin{vmatrix} b_j \\ b_j \end{vmatrix} \tag{1.5}$$

where Z is an $n \times n$ matrix with all zero entries and I is the $n \times n$ unit matrix. By substituting the expression (1.5) in the right-hand member of (1.1) and carrying out the blocked-matrix product, we obtain the $2n \times 2n$ matricial equation

$$\begin{vmatrix} b_i \\ a_i \end{vmatrix} = \begin{vmatrix} T_1 & T_2 \\ T_3 & T_4 \end{vmatrix} \cdot \begin{vmatrix} S_X & Z \\ Z & I \end{vmatrix} \cdot \begin{vmatrix} b_j \\ b_j \end{vmatrix} = \begin{vmatrix} T_1 \cdot S_X & T_2 \\ T_3 \cdot S_X & T_4 \end{vmatrix} \cdot \begin{vmatrix} b_j \\ b_j \end{vmatrix} \quad (1.6)$$

which may be split horizontally in the two $n \times n$ matricial equations

$$|b_i| = (T_1 \cdot S_X + T_2) \cdot |b_j| \quad (1.7)$$

$$|a_i| = (T_3 \cdot S_X + T_4) \cdot |b_j|. \quad (1.8)$$

Equation (1.8) may be solved for the vector b_j by premultiplication by the matrix $(T_3 \cdot S_X + T_4)^{-1}$, obtaining

$$|b_j| = (T_3 \cdot S_X + T_4)^{-1} \cdot |a_i|. \quad (1.9)$$

Finally, by substituting (1.9) in (1.7) we have

$$|b_i| = (T_1 \cdot S_X + T_2) \cdot (T_3 \cdot S_X + T_4)^{-1} \cdot |a_i| \quad (1.10)$$

and, comparing it with the definition $|b_i| = S_M \cdot |a_i|$ of the measured, input-interface $n \times n$ scattering matrix S_M, we have

$$S_M = (T_1 \cdot S_X + T_2) \cdot (T_3 \cdot S_X + T_4)^{-1}. \quad (1.11)$$

This equation shows how the four quadrants $T_1, \cdots, T_4$ of the error-network T-matrix relate the measured scattering matrix S_M to the true scattering matrix S_X of an unknown network X. For an arbitrary calibration standard i, the same equation may be rewritten as

$$S_{Mi} = (T_1 \cdot S_{Si} + T_2) \cdot (T_3 \cdot S_{Si} + T_4)^{-1} \quad (1.12)$$

which is the form given as (1) of the text.

Appendix II

The fundamental Super-TSD calibration equations, given by (2) of Section II-D, may be explicitly rewritten for three arbitrary calibration standards, $i = 1, 2, 3$, as

$$(I \otimes S_{S1}^T) \cdot RS(T_1) + RS(T_2) - (S_{M1} \otimes S_{S1}^T) \cdot RS(T_3) - (S_{M1} \otimes I) \cdot RS(T_4) = |0| \quad (2.1)$$

$$(I \otimes S_{S2}^T) \cdot RS(T_1) + RS(T_2) - (S_{M2} \otimes S_{S2}^T) \cdot RS(T_3) - (S_{M2} \otimes I) \cdot RS(T_4) = |0| \quad (2.2)$$

$$(I \otimes S_{S3}^T) \cdot RS(T_1) + RS(T_2) - (S_{M3} \otimes S_{S3}^T) \cdot RS(T_3) - (S_{M3} \otimes I) \cdot RS(T_4) = |0| \quad (2.3)$$

where the $|0|$'s in the right-hand members are zero vectors of order n^2. Then, by subtracting (2.2) from (2.1), we have

$$[(I \otimes S_{S1}^T) - (I \otimes S_{S2}^T)] \cdot RS(T_1) - [(S_{M1} \otimes S_{S1}^T) - (S_{M2} \otimes S_{S2}^T)] \cdot RS(T_3) - [(S_{M1} \otimes I) - (S_{M2} \otimes I)] \cdot RS(T_4) = E \cdot RS(T_1) - A \cdot RS(T_3) - B \cdot RS(T_4) = |0| \quad (2.4)$$

and, by subtracting (2.3) from (2.1), we have

$$[(I \otimes S_{S1}^T) - (I \otimes S_{S3}^T)] \cdot RS(T_1) - [(S_{M1} \otimes S_{S1}^T) - (S_{M3} \otimes S_{S3}^T)] \cdot RS(T_3) - [(S_{M1} \otimes I) - (S_{M3} \otimes I)] \cdot RS(T_4) = F \cdot RS(T_1) - C \cdot RS(T_3) - D \cdot RS(T_4) = |0|. \quad (2.5)$$

Also, by premultiplying (2.4) by A^{-1} and (2.5) by C^{-1}, we have

$$RS(T_3) + A^{-1} \cdot B \cdot RS(T_4) = A^{-1} \cdot E \cdot RS(T_1) \quad (2.4')$$

$$RS(T_3) + C^{-1} \cdot D \cdot RS(T_4) = C^{-1} \cdot F \cdot RS(T_1) \quad (2.5')$$

and subtracting (2.5′) from (2.4′)

$$(A^{-1}B - C^{-1}D) \cdot RS(T_4) = (A^{-1}E - C^{-1}F) \cdot RS(T_1) \quad (2.6)$$

which may be solved for the vector $RS(T_4)$

$$RS(T_4) = (A^{-1}B - C^{-1}D)^{-1} \cdot (A^{-1}E - C^{-1}F) \cdot RS(T_1). \quad (2.6')$$

Similarly, by premultiplying (2.4) by B^{-1} and (2.5) by D^{-1}, we have

$$B^{-1} \cdot A \cdot RS(T_3) + RS(T_4) = B^{-1} \cdot E \cdot RS(T_1) \quad (2.4'')$$

$$D^{-1} \cdot C \cdot RS(T_3) + RS(T_4) = D^{-1} \cdot F \cdot RS(T_1) \quad (2.5'')$$

and subtracting (2.5″) from (2.4″)

$$(B^{-1}A - D^{-1}C) \cdot RS(T_3) = (B^{-1}E - D^{-1}F) \cdot RS(T_1) \quad (2.7)$$

which may be solved for the vector $RS(T_3)$

$$RS(T_3) = (B^{-1}A - D^{-1}C)^{-1} \cdot (B^{-1}E - D^{-1}F) \cdot RS(T_1). \quad (2.7')$$

Finally, by substituting (2.6′) and (2.7′) in (2.1), we have

$$RS(T_2) = [(S_{M1} \otimes S_{S1}^T) \cdot (B^{-1}A - D^{-1}C)^{-1} \cdot (B^{-1}E - D^{-1}F) + (S_{M1} \otimes I) \cdot (A^{-1}B - C^{-1}D)^{-1} \cdot (A^{-1}E - C^{-1}F) - (I \otimes S_{S1}^T)] \cdot RS(T_1). \quad (2.8)$$

The obtained equations (2.6′), (2.7′), and (2.8) represent the explicit solution of the Super-TSD calibration equations, as already given by (4)–(6) in Section II-D.

Following a procedure quite similar to that outlined by (2.1)–(2.8), it is also possible to directly compute the quadrants $R_1, \cdots, R_4$ of the inverse $R = T^{-1}$ of the error network appearing in the first form of (13) of Section II-E. It is convenient here to express these R_i quadrants in terms of their $S(R_i)$ stacking operators.

The final solution is given by

$$S(R_1) = \text{Arbitrary nonzero complex column vector of order } n^2 \tag{2.9}$$

$$S(R_2) = -(B_2^{-1}E_3 - D_2^{-1}F_3)^{-1} \cdot (B_2^{-1}A_3 - D_2^{-1}C_3) \cdot S(R_1) \tag{2.10}$$

$$S(R_3) = (E_2^{-1}A_2 - F_2^{-1}C_2)^{-1} \cdot (E_2^{-1}B_2 - F_2^{-1}D_2) \cdot S(R_1) \tag{2.11}$$

$$S(R_4) = (A_2^{-1}E_2 - C_2^{-1}F_2)^{-1} \cdot (A_2^{-1}B_2 - C_2^{-1}D_2) \cdot S(R_1) \tag{2.12}$$

where the auxiliary functions $A_2, \cdots, F_2$ and A_3, C_3, E_3, and F_3 are expressed by

$$A_2 = (S_{M1}^T \otimes S_{S1}) - (S_{M2}^T \otimes S_{S2}) \tag{2.13}$$

$$B_2 = (S_{M1}^T \otimes I) - (S_{M2}^T \otimes I) \tag{2.14}$$

$$C_2 = (S_{M1}^T \otimes S_{S1}) - (S_{M3}^T \otimes S_{S3}) \tag{2.15}$$

$$D_2 = (S_{M1}^T \otimes I) - (S_{M3}^T \otimes I) \tag{2.16}$$

$$E_2 = (I \otimes S_{S1}) - (I \otimes S_{S2}) \tag{2.17}$$

$$F_2 = (I \otimes S_{S1}) - (I \otimes S_{S3}) \tag{2.18}$$

$$A_3 = (S_{M1}^T \otimes S_{S1}^{-1}) - (S_{M2}^T \otimes S_{S2}^{-1}) \tag{2.19}$$

$$C_3 = (S_{M1}^T \otimes S_{S1}^{-1}) - (S_{M3}^T \otimes S_{S3}^{-1}) \tag{2.20}$$

$$E_3 = (I \otimes S_{S1}^{-1}) - (I \otimes S_{S2}^{-1}) \tag{2.21}$$

$$F_3 = (I \otimes S_{S1}^{-1}) - (I \otimes S_{S3}^{-1}). \tag{2.22}$$

Appendix III

By solving the $2n \times 2n$ matrix equation (1.1) for the vector of the output-interface waves a_j, b_j, we have

$$\begin{vmatrix} a_j \\ b_j \end{vmatrix} = T^{-1} \cdot \begin{vmatrix} b_i \\ a_i \end{vmatrix} = \begin{vmatrix} T_1 & T_2 \\ T_3 & T_4 \end{vmatrix}^{-1} \cdot \begin{vmatrix} b_i \\ a_i \end{vmatrix} = \begin{vmatrix} R_1 & R_2 \\ R_3 & R_4 \end{vmatrix} \cdot \begin{vmatrix} b_i \\ a_i \end{vmatrix} \tag{3.1}$$

while, by definition, we have

$$|b_i| = S_M \cdot |a_i| \tag{3.2}$$

or

$$\begin{vmatrix} b_i \\ a_i \end{vmatrix} = \begin{vmatrix} S_M & Z \\ Z & I \end{vmatrix} \cdot \begin{vmatrix} a_i \\ a_i \end{vmatrix} \tag{3.3}$$

where Z and I are again the $n \times n$ zero and unit matrices, respectively. By substituting the expression (3.3) of the input-interface wave vector in (3.1), we obtain the $2n \times 2n$ matricial equation

$$\begin{vmatrix} a_j \\ b_j \end{vmatrix} = \begin{vmatrix} R_1 & R_2 \\ R_3 & R_4 \end{vmatrix} \cdot \begin{vmatrix} S_M & Z \\ Z & I \end{vmatrix} \cdot \begin{vmatrix} a_i \\ a_i \end{vmatrix} = \begin{vmatrix} R_1 \cdot S_M & R_2 \\ R_3 \cdot S_M & R_4 \end{vmatrix} \cdot \begin{vmatrix} a_i \\ a_i \end{vmatrix} \tag{3.4}$$

which may be split horizontally in the two $n \times n$ matricial equations

$$|a_j| = (R_1 \cdot S_M + R_2) \cdot |a_i| \tag{3.5}$$

$$|b_j| = (R_3 \cdot S_M + R_4) \cdot |a_i|. \tag{3.6}$$

Equation (3.6) may then be solved for the vector a_i of the input-interface incident waves

$$|a_i| = (R_3 \cdot S_M + R_4)^{-1} \cdot |b_j| \tag{3.7}$$

and by substituting this expression in (3.5) we have

$$|a_j| = (R_1 \cdot S_M + R_2) \cdot (R_3 \cdot S_M + R_4)^{-1} \cdot |b_j| \tag{3.8}$$

which, compared with the definition of the $n \times n$ scattering matrix of the unknown network given by (1.4), provides

$$S_X = (R_1 \cdot S_M + R_2) \cdot (R_3 \cdot S_M + R_4)^{-1}. \tag{3.9}$$

The quadrants $R_1, \cdots, R_4$ of the inverse T^{-1} of the T-parameter matrix T of the error network may also be expressed by [24, p. 92, problem 4]

$$R_1 = (T_1 - T_2 T_4^{-1} T_3)^{-1} \tag{3.10}$$

$$R_2 = (T_3 - T_4 T_2^{-1} T_1)^{-1} \tag{3.11}$$

$$R_3 = (T_2 - T_1 T_3^{-1} T_4)^{-1} \tag{3.12}$$

$$R_4 = (T_4 - T_3 T_1^{-1} T_2)^{-1}. \tag{3.13}$$

By substituting these expressions of the $n \times n$ quadrants of the $2n \times 2n$ inverse T-matrix in (3.9), the n-port deembedment equation given in Section II-E is obtained.

It may also be shown that if the quadrants $T_1, \cdots, T_4$ of the matrix T commute ($T_i T_j = T_j T_i$ for $i \neq j$), then (3.9) reduces to

$$S_X = (T_4 \cdot S_M - T_2) \cdot (-T_3 \cdot S_M + T_1)^{-1}.$$

This is in particular true in the zero-leakage case where the quadrants $T_1, \cdots, T_4$ are diagonal matrices.

Appendix IV

The S to S^* transformation (29)–(32) may easily be obtained by substituting the S to $ABCD$ transformation into the $ABCD$ to S^* transformation.

The chain parameters $ABCD$ of a network may be expressed as functions of its standard S-parameters (normalized to a real Z_0) as

$$\begin{aligned} A &= \frac{1}{2S_{21}}[(1+S_{11})(1-S_{22}) + S_{12}S_{21}] \\ &= \frac{1}{2S_{21}}\{[1 - \text{DET }(S)] + (S_{11} - S_{22})\} \\ &= \frac{X+Y}{2S_{21}} \end{aligned} \tag{4.1}$$

$$\begin{aligned} B &= \frac{1}{2S_{21}}[(1+S_{11})(1+S_{22}) - S_{12}S_{21}]Z_0 \\ &= \frac{1}{2S_{21}}\{[1 + \text{DET }(S)] + (S_{11} + S_{22})\}Z_0 \\ &= \frac{U+V}{2S_{21}}Z_0 \end{aligned} \tag{4.2}$$

$$C = \frac{1}{2S_{21}}[(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}]\frac{1}{Z_0}$$
$$= \frac{1}{2S_{21}}\{[1 + \text{DET }(S)] - (S_{11} + S_{22})\}\frac{1}{Z_0}$$
$$= \frac{U - V}{2S_{21}}\frac{1}{Z_0} \tag{4.3}$$

$$D = \frac{1}{2S_{21}}[(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}]$$
$$= \frac{1}{2S_{21}}\{[1 - \text{DET }(S)] - (S_{11} - S_{22})\}$$
$$= \frac{X - Y}{2S_{21}} \tag{4.4}$$

while

$$AD - BC = \frac{S_{12}}{S_{21}}. \tag{4.5}$$

The supergeneralized S^*-parameters, normalized to the complex port impedances Z_{nij} and related to traveling waves, may be expressed as functions of the $ABCD$ parameters as

$$S^*_{11} = \frac{A\sqrt{\frac{Z_{211}}{Z_{111}}} + \frac{B}{\sqrt{Z_{111}Z_{211}}} - C\sqrt{Z_{111}Z_{211}} - D\sqrt{\frac{Z_{111}}{Z_{211}}}}{A\sqrt{\frac{Z_{211}}{Z_{111}}} + \frac{B}{\sqrt{Z_{111}Z_{211}}} + C\sqrt{Z_{111}Z_{211}} + D\sqrt{\frac{Z_{111}}{Z_{211}}}} \tag{4.6}$$

$$S^*_{12} = \frac{2(AD - BC)}{A\sqrt{\frac{Z_{212}}{Z_{112}}} + \frac{B}{\sqrt{Z_{112}Z_{212}}} + C\sqrt{Z_{112}Z_{212}} + D\sqrt{\frac{Z_{112}}{Z_{212}}}} \tag{4.7}$$

$$S^*_{21} = \frac{2}{A\sqrt{\frac{Z_{221}}{Z_{121}}} + \frac{B}{\sqrt{Z_{121}Z_{221}}} + C\sqrt{Z_{121}Z_{221}} + D\sqrt{\frac{Z_{121}}{Z_{221}}}} \tag{4.8}$$

$$S^*_{22} = -\frac{A\sqrt{\frac{Z_{222}}{Z_{122}}} - \frac{B}{\sqrt{Z_{122}Z_{222}}} + C\sqrt{Z_{122}Z_{222}} - D\sqrt{\frac{Z_{122}}{Z_{222}}}}{A\sqrt{\frac{Z_{222}}{Z_{122}}} + \frac{B}{\sqrt{Z_{122}Z_{222}}} + C\sqrt{Z_{122}Z_{222}} + D\sqrt{\frac{Z_{122}}{Z_{222}}}}. \tag{4.9}$$

If the S^*-parameters related to Youla's power waves [25], [26] are desired, then (4.1)–(4.5) should be substituted in the $ABCD$ to power-wave S^*-transformation, given by Chen in [27, p. 436, eq. (12c)]. Chen's z_i and k_i parameters and the denominator q_3 must, however, be changed for every element of the $S(p)$ matrix.

Appendix V

The expressions reported here summarize the explicit solution of the TSD calibration equations in a form slightly different than the one given in the original report [19]. In particular, the definitions given here of the fundamental T-matrix products H and K are the new definitions introduced in [23] and discussed in Section III-D.

The three two-port calibration standards used are the Through, defined as a residual length L_1 of nominal-impedance transmission line; the Short, defined as a pair of immediate shorts at both measurement ports; and the Delay, defined as a substantial length L_2 of nominal-impedance transmission line.

The need for selecting among multiple intermediate solutions, at each frequency point, motivates the assumption of the length of the line L_1 being less than $\frac{1}{4}\lambda$ at any frequency. The lines L_1 and L_2 could be segments of dispersive waveguide with the same cross-section geometry and cutoff frequency.

In contrast to Super-TSD, where all calibration standards are assumed to be fully known, the electrical lengths of L_1 and L_2 need not to be accurately known in TSD, as they are computed in the process together with the respective insertion losses.

The postulated and measured scattering matrices of the three standards S_{Si} and S_{Mi} are expressed as

$$S_{S1} = S_{ST} = \begin{vmatrix} 0 & e^{-\rho_1} \\ e^{-\rho_1} & 0 \end{vmatrix} \tag{5.1}$$

$$S_{S2} = S_{SS} = \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} \tag{5.2}$$

$$S_{S3} = S_{SD} = \begin{vmatrix} 0 & e^{-\rho_2} \\ e^{-\rho_2} & 0 \end{vmatrix} \tag{5.3}$$

$$S_{M1} = S_T = \begin{vmatrix} S_{11T} & S_{12T} \\ S_{21T} & S_{22T} \end{vmatrix} \tag{5.4}$$

$$S_{M2} = S_S = \begin{vmatrix} S_{11S} & 0 \\ 0 & S_{22S} \end{vmatrix} \tag{5.5}$$

$$S_{M3} = S_D = \begin{vmatrix} S_{11D} & S_{12D} \\ S_{21D} & S_{22D} \end{vmatrix} \tag{5.6}$$

where $\rho_1 = r_1 + j\theta_1$ and $\rho_2 = r_2 + j\theta_2$. Consistent with the zero-leakage assumption, the transmission entries S_{12S} and

S_{21S} measured for the Short are assumed to be identically zero.

The "measured" Tmatrices of the Through $T_T = T_{D1}$ and of the Delay $T_D = T_{D2}$ correspond to the S-matrices S_T and S_D and are computed from these through the standard S to T-parameter transformation. The unknown scattering matrices of the error two-ports A and B are expressed as

$$S_A = \begin{vmatrix} S_{11A} & S_{12A} \\ S_{21A} & S_{22A} \end{vmatrix} \quad (5.7)$$

$$S_B = \begin{vmatrix} S_{11B} & S_{12B} \\ S_{21B} & S_{22B} \end{vmatrix}. \quad (5.8)$$

The fundamental T-matrix products H and K are built according to the new definitions as

$$H = e^{-\delta} T_{D1} T_{D2}^{-1} \quad (5.9)$$

$$K = e^{-\delta} T_{D2}^{-1} T_{D1}. \quad (5.10)$$

The eight elements of these two matrix products may be directly expressed as scalar complex functions of the elements of the S_T and S_D matrices, as follows:

$$H_{11} = R[S_{11T}S_{22D} - \text{DET}\,(S_T)] \quad (5.11)$$

$$H_{12} = R[S_{11D}\,\text{DET}\,(S_T) - S_{11T}\,\text{DET}\,(S_D)] \quad (5.12)$$

$$H_{21} = -R(S_{22T} - S_{22D}) \quad (5.13)$$

$$H_{22} = R[S_{11D}S_{22T} - \text{DET}\,(S_D)] \quad (5.14)$$

$$K_{11} = R[S_{11D}S_{22T} - \text{DET}\,(S_T)] \quad (5.15)$$

$$K_{12} = R(S_{11T} - S_{11D}) \quad (5.16)$$

$$K_{21} = R[S_{22T}\,\text{DET}\,(S_D) - S_{22D}\,\text{DET}\,(S_T)] \quad (5.17)$$

$$K_{22} = R[S_{11T}S_{22D} - \text{DET}\,(S_D)] \quad (5.18)$$

where

$$R = \frac{1}{\sqrt{S_{12T}S_{21T}S_{12D}S_{21D}}}. \quad (5.19)$$

The auxiliary functions $H_1, \cdots, H_4$ and $K_1, \cdots, K_4$ are then defined as

$$H_1 = \tfrac{1}{2}[R'(H_{22} - H_{11}) + 1] \quad (5.20)$$

$$H_2 = \tfrac{1}{2}[R'(H_{22} - H_{11}) - 1] \quad (5.21)$$

$$H_3 = R' \cdot H_{12} \quad (5.22)$$

$$H_4 = -R' \cdot H_{21} \quad (5.23)$$

$$K_1 = \tfrac{1}{2}[R'(K_{22} - K_{11}) + 1] \quad (5.24)$$

$$K_2 = \tfrac{1}{2}[R'(K_{22} - K_{11}) - 1] \quad (5.25)$$

$$K_3 = -R' \cdot K_{12} \quad (5.26)$$

$$K_4 = R' \cdot K_{21} \quad (5.27)$$

where

$$R' = \frac{1}{\sqrt{(H_{11} + H_{22})^2 - 4}} = \frac{1}{\sqrt{(K_{11} + K_{22})^2 - 4}}. \quad (5.28)$$

On the basis of these auxiliary functions, the S-matrices S_A and S_B are given by either of the following two solutions.

First Solution:

$$S_{11A} = \frac{H_3}{H_1} = \frac{H_2}{H_4} \quad (5.29)$$

$$S_{22A} = \frac{\dfrac{H_3}{H_1} - S_{11S}}{S_{11S} - \dfrac{H_1}{H_4}} \quad (5.30)$$

$$S_{12A}S_{21A} = -\frac{S_{22A}}{H_4} \quad (5.31)$$

$$\text{DET}\,(S_A) = \frac{H_1}{H_4}S_{22A} = \frac{H_3}{H_2}S_{22A} \quad (5.32)$$

$$S_{11B} = -\frac{S_{22S} + \dfrac{K_4}{K_1}}{S_{22S} + \dfrac{K_1}{K_3}} \quad (5.33)$$

$$S_{22B} = -\frac{K_4}{K_1} = -\frac{K_2}{K_3} \quad (5.34)$$

$$S_{12B}S_{21B} = \frac{S_{11B}}{K_3} \quad (5.35)$$

$$\text{DET}\,(S_B) = -\frac{K_1}{K_3}S_{11B} = -\frac{K_4}{K_2}S_{11B}. \quad (5.36)$$

Second Solution:

$$S_{11A} = \frac{H_3}{H_2} = \frac{H_1}{H_4} \quad (5.37)$$

$$S_{22A} = \frac{S_{11S} - \dfrac{H_1}{H_4}}{\dfrac{H_3}{H_1} - S_{11S}} \quad (5.38)$$

$$S_{12A}S_{21A} = \frac{S_{22A}}{H_4} \quad (5.39)$$

$$\text{DET}\,(S_A) = \frac{H_2}{H_4}S_{22A} = \frac{H_3}{H_1}S_{22A} \quad (5.40)$$

$$S_{11B} = -\frac{S_{22S} + \dfrac{K_1}{K_3}}{S_{22S} + \dfrac{K_4}{K_1}} \quad (5.41)$$

$$S_{22B} = -\frac{K_1}{K_3} = -\frac{K_4}{K_2} \quad (5.42)$$

$$S_{12B}S_{21B} = -\frac{S_{11B}}{K_3} \quad (5.43)$$

$$\text{DET}\,(S_B) = -\frac{K_4}{K_1}S_{11B} = -\frac{K_2}{K_3}S_{11B}. \quad (5.44)$$

The existence of two alternative solutions is a consequence of the unique TSD aspect of having the elements $e^{-\rho_1}$ and $e^{-\rho_2}$ of the postulated S-matrices S_T and S_D playing the roles of "associated unknowns" of the problem. The two alternative TSD solutions correspond to mutually reciprocal and opposite values for either of these matrix elements.

Some kind of flag-function is then required to select, at every frequency, the solution corresponding to a physically meaningful situation with the electrical lengths of both lines L_1 and L_2 positive. In these respects, by working out the T-matrix product $T_{L1} = T_A^{-1} T_T T_B^{-1}$, which is equivalent to stripping the computed error two-ports A and B from the through, expressions are obtained of the two nonzero elements of T_{L1}

$$T_{11L1} = e^{-\rho_1} = \frac{1}{S_{12A} S_{12B} S_{21T}} \cdot [S_{11A} S_{22T} - \mathrm{DET}\,(S_T) + (S_{11T} - S_{11A}) S_{22B}] \tag{5.45}$$

$$T_{22L1} = e^{\rho_1} = -\frac{1}{S_{12A} S_{12B} S_{21T}} \cdot \{S_{11B}[S_{22T}\,\mathrm{DET}\,(S_A) - S_{22A}\,\mathrm{DET}\,(S_T)] + [S_{11T} S_{22A} - \mathrm{DET}\,(S_A)]\,\mathrm{DET}\,(S_B)\}. \tag{5.46}$$

These two expressions have been shown to be mutually reciprocal under both solutions. Their ratio, however, expressed by

$$e^{2\rho_1} = \frac{e^{\rho_1}}{e^{-\rho_1}} = S_{11B} S_{22A} \frac{S_{22T} - \dfrac{\mathrm{DET}\,(S_B)}{S_{11B}}}{S_{22T} - S_{22B}} \tag{5.47}$$

assumes two mutually reciprocal values under the two alternative TSD solutions so that its imaginary part may be used as the required flag-function, signaling which of the two solutions is physically meaningful. Indeed, if the electrical length of the line L_1 is assumed to be always $< \frac{1}{4}\lambda$ (it is a "residual" length in real life!), then the imaginary part of $e^{2\rho_1}$ must always be positive.

It is impossible, on physical grounds, to separate S_{12A} from S_{21A} and S_{12B} from S_{21B}. If, however, we accept the arbitrary assumption

$$\frac{S_{12A}}{S_{21A}} = \frac{S_{12B}}{S_{21B}} = \sqrt{\frac{S_{12T}}{S_{21T}}} \tag{5.48}$$

separate values of the $S_{ij}\ (i \neq j)$ may be computed. Assumption (5.48) forces the error two-ports A and B to share in equal proportions the apparent nonreciprocity due to the system errors.

References

[1] R. A. Hackborn, "An automatic network analyzer system," *Microwave J.*, vol. 11, pp 45–52, May 1968.

[2] B. P. Hand, "Developing accuracy specifications for automatic network analyzer systems," *Hewlett-Packard J.*, vol. 21, pp. 16–19, Feb. 1970.

[3] J. D. Last and S. T. Smith, "Offline correction of microwave network analyser reflection-coefficient errors," *Electron. Lett.*, vol. 6, p. 641, Oct. 1, 1970.

[4] H. V. Shurmer, "New method of calibrating a network analyser," *Electron. Lett.*, vol. 6, pp. 733–734, Oct. 12, 1970.

[5] W. Kruppa and K. F. Sodomski, "An explicit solution for the scattering parameters of a linear two-port measured with an imperfect test set," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 122–123, Jan. 1971

[6] H. V. Shurmer, "Low-level programming for the on-line correction of microwave measurements," *Radio Electron. Eng.*, vol. 41, pp. 357–364, Aug. 1971.

[7] C. T. M. Chang, "Application of Deschamp's graphical method to measurements of the scattering coefficients of multiport waveguide junctions," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 186–187, Feb. 1972.

[8] V. Riech, "Measurement program and calibration standards for microstrip measurements with an automatic network analyzer," *Nachrichtentech. Z*, vol. 24, pp. 255–259, May 1971.

[9] E. F. DaSilva and M. K. McPhun, "Calibration of microwave network analyser for computer-corrected S-parameter measurements," *Electron. Lett.*, vol. 9, pp. 126–128, Mar. 22, 1973.

[10] H. V. Shurmer, "Calibration procedure for computer-corrected S-parameter characterization of devices mounted in microstrip," *Electron. Lett.*, vol. 9, pp. 323–324, July 12, 1973.

[11] J W. Gould and G. M. Rhodes, "Computer correction of a microwave network analyser without accurate frequency measurement," *Electron. Lett.*, vol. 9, pp. 494–495, Oct. 18, 1973.

[12] O J. Davies, R. B. Doshi, and B. Nagenthiram, "Correction of microwave-network-analyser measurements of 2-port devices," *Electron. Lett.*, vol 9, pp. 543–544, Nov. 15, 1973.

[13] S. Rehnmark, "On the calibration process of automatic network analyzer systems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 457–458, Apr 1974.

[14] I. Kasa, "Exact solution of network analyzer calibration and two-port measurements by sliding terminations," in *Dig. Tech. Papers 1974 Conf. Precision Electromagnetic Measurements*, July 1–5, 1974, pp. 90–92.

[15] ——, "Closed-form mathematical solutions to some network analyzer calibration equations," *IEEE Trans. Instrum Meas*, vol. IM-23, pp. 399–402, Dec. 1974.

[16] G. F. Engen, "An alternative calibration technique for automated network analyzers with application to adapter evaluation," in *Dig. Tech. Papers 1974 IEEE MTT-S Int. Microwave Symp.* (Atlanta, GA), June 12–14, 1974, pp 261–262.

[17] ——, "Calibration technique for automated network analyzers with application to adapter evaluation," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 1255–1260, Dec. 1974.

[18] D. Woods, "Rigorous derivation of computer corrected network analyser calibration equations," *Electron. Lett.*, vol 11, pp. 403–404, Aug. 21, 1975.

[19] N. R. Franzen and R. A. Speciale, "A new procedure for system calibration and error removal in automated S-parameter measurements," in *Proc. 5th European Microwave Conf.* (Hamburg, Germany), Sept. 1–4, 1975, pp. 69–73.

[20] V. G. Gelnovatch, "A computer program for the direct calibration of two-port reflectometers for automated microwave measurements," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 45–47, Jan. 1976.

[21] R. A. Speciale, "Automated component testing—It's a reality .. Thanks to minicomputers," *Microwave Syst. News*, vol. 6, pp. 79–99, June/July 1976.

[22] N. R. Franzen and R. A. Speciale, "Accurate scattering parameter measurements on non-connectable microwave networks," in *Proc. 6th European Microwave Conf.* (Rome, Italy), Sept. 14–17, 1976, pp. 210–214.

[23] ——, "Super-TSD. A generalization of the TSD network analyzer calibration procedure, covering n-port measurements with leakage," in *1977 IEEE-MTT-S Int. Microwave Symp., Dig Tech. Papers* (San Diego, CA), June 21–23, 1977, pp. 114–117.

[24] R. Bellman, *Introduction to Matrix Analysis*, 2nd ed. New York: McGraw-Hill, 1970, pp. 235–236; also p. 245, problem 16, and p. 92, problem 4

[25] D. C. Youla, "On scattering matrices normalized to complex port numbers," *Proc. IRE*, vol. 49, p. 1221, July 1961.

[26] K. Kurokawa, "Power waves and the scattering matrix," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 194–202, Mar. 1965

[27] W. K Chen, "Relationship between scattering matrix and other matrix representations of linear two-port networks," *Int. J. Electron.*, vol. 38, pp. 433–441, Apr. 1975.